Pearson Edexcel

Examiners' Report
Principal Examiner Feedback

January 2024

Pearson Edexcel International Advanced Level in Pure Mathematics P4 (WMA14) Paper 01

WMA14 January 2024 Examiners Report

General

This was an accessible WMA14 paper with little evidence of students failing to complete the entire paper. There was plenty of access in the early questions, with some challenges later in the paper to test the best of students. There were fewer blank attempts at questions in this series than in some previous ones, though there was evidence of under-prepared students in some scripts. Generally, the standard of algebra was sound though the importance of showing all working should be stressed.

Individual Question report

Question 1

In general students found this to be an accessible question, and the majority were able to score full marks. Very few students were unable to make a start to their answer.

Most were able to successfully apply the correct method for finding a binomial expansion, reaching the correct simplified answer. Where errors did occur, they were usually the result of using 'x' or '$4x$' in the place of '$-4x$' in the expansion, but in these cases some marks were still scored. There were a few instances of numerical slips, and sometimes the constant term was omitted, but these situations were rare. Bracketing errors were also comparatively rare.

Just a few students wrote the expression as $\frac{1}{(1-4x)^3}$ and expanded this denominator, scoring no marks.

Question 2

Overall this question was done quite well, with almost all students making progress in both parts. The methodology of part (a) was particularly well known.

In part (a) the majority of students multiplied the fractions correctly, enabling them to compare the numerators. Most went directly to the numerators, without showing the full identity. Relatively few students made the error of multiplying by the product of the three denominators so having $(2x + 1)^3$. The most common method was by substitution, $x = 2$ and $x = -\frac{1}{2}$, and usually $x = 0$, to find B. A few students compared coefficients for B, having used substitution for A and C. There were some arithmetical errors, but overall there was a good success rate using this method. A significant number of students instead expanded the numerator and used simultaneous equations to find the values of A, B and C. Although many were successful, this method was more prone to error than attempts via substitution. Overall a well attempted and accessible part.

Pleasingly, most students continued to make a good attempt at part (b), achieving at least 3 of the method marks. Only a few failed to make any progress. Most students were able to attain the first method mark by integrating the first term to the correct form, with most having the integration correct. Although some responses used $5x – 10$ in the denominator, most left the $\frac{2}{5}$ outside and integrated to $\frac{2}{5}\ln(x-2)$. Use of $5x – 10$ did result in more errors. Integration of the second and third terms was less successful, though the ln form was usually achieved. A number of students failed to divide the second logarithm by two and this caused problems when collecting the terms after substitution. Although a high proportion of students did correctly integrate $(2x+1)^{-2}$ to the correct form, there were quite a lot of errors with the sign and the coefficient. Also, a natural logarithm was seen frequently for the final term. The mark for substitution was attained by most students and some evidence of correct use at least one of the laws of logarithms was also seen in most responses, though the ability to attain a single logarithm depended to a great extent on the coefficients attained. Many students simply put their coefficients as powers and ended up with a single logarithm which looked very messy, while others were unable to use the power law correctly, or ignored the leading coefficients and such errors. Those who combined logarithms before substitution were generally more successful.

Question 3

This question was well attempted with many students obtaining full marks.

In part (a), the majority of students used implicit differentiation correctly and then rearranged to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. Where errors did occur, they were usually a result of differentiating y^2x incorrectly using the product rule, and a common wrong answer was $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8x-y^2}{2y+3}$, occasionally obtained by misreading an x as a multiplication sign, or missing the x entirely throughout, and sometimes with the missing x present until the factorisation. Some students only obtained one term as a result of the product rule, differentiating y^2x to just $2y\frac{\mathrm{d}y}{\mathrm{d}x}$, while on the other extreme some obtained an incorrect term of $y^2\frac{\mathrm{d}y}{\mathrm{d}x}$, usually resulting in three terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$ and hence in the denominator, leaving a numerator of just $8x$.

The remaining integration of $3y \to 3\frac{dy}{dx}$ and $4x^2 \to 8x$ was usually successfully carried out with occasional sight of just '3' or '$3y\frac{dy}{dx}$. Also, the '+k' term sometimes incorrectly appeared in the differentiated expression. Few students made no attempt at differentiation, or made an entirely incorrect attempt. There were also very few cases of an extra $\frac{dy}{dx} =$ being used.

In part (b) students generally knew how to progress, and used $\frac{dy}{dx} = 0$, as well as the original equation with the point (p, 2) to find values of p and k. There were a few instances of numerical errors with $p = 2$ being a common incorrect answer (following $8p = 4$). Those whose derivative included the k usually struggled to make progress in this part. A small number of students used the alternative method for (b) and solved successfully without use of the derivative, but such cases were rare.

Question 4

This question proved challenging to a very large majority of students. The concept of rates of change was shown to be not well understood by most, although the use of the chain rule was usually demonstrated.

In part (a), the majority were able to make the connection between l, r and the height, 5, and thus found a correct expression for l in terms of r, using Pythagoras' theorem. Nevertheless there were a significant number making errors even at this stage. The most common was applying Pythagoras' theorem incorrectly to $l^2 = 25 - r^2$, or with terms switched. A small number expressed r in terms of l, or simply left the answer as an expression for l^2. It was, perhaps, rather concerning - for students studying at this level - that too many, after stating $l = \sqrt{r^2 + 25}$, went on to write $l = r + 5$. After a correct response, such material was ignored. Fully correct responses to part (b) were rare, with many only able to score the second method mark. A failure to realise use the result of (a) was the most common error, with many students treating l in the given $S = \pi.r$^2 + $\pi.r.l$ as a constant when attempting to find $\frac{dS}{dr}$ and making little or no further progress. Although the more capable students could substitute their expression for l from part (a) into the above formula for S, errors in differentiating (using the product rule) were not uncommon. However, most students were able to successfully use the chain rule to find $\frac{dS}{dt}$ via $\frac{dS}{dt} = \frac{dS}{dr}\frac{dr}{dt}$, by substituting $r = 1.5$ in their $\frac{dS}{dr}$ expression and multiplying by 3, the given value of $\frac{dr}{dt}$, thus scoring at least one method mark in this part of the question. Some very astute students proceeded via the alternative methods, using implicit differentiation or related methods.

Question 5

Another question that gave good access, with part (a) by and large successfully complete and the separation of variables and integration of the left hand side both well attempted in part (b).

Many students scored all four marks for part (a) with the method for integration by parts seeming well drilled into students. A few used the tabular "DI" approach, but the majority opted for the more traditional method, showing the two stages of parts being used. Only a very small number failed to attempt integration by parts at all, and most applied a correct formula. Where errors were seen they were usually sign errors in the second stage of integration by parts. The first stage usually carried out correctly, though occasionally the constants and/or the signs in the either/both terms were incorrect. Very occasionally the first part (uv) was missing an 'x', i.e. given as $Ax \sin 2x$, or one/both terms were given in terms of '$\cos 2x$'.

If the first method mark was scored, students usually continued correctly to score the second by attempting parts again in the same direction, but errors in signs were not uncommon at this stage, particularly in cases where the integration by parts had been attempted as separate pieces of work and then combined, 'losing negatives'.

In part (b), the separation of variables was generally successfully attempted, though a few were unable to take the y term across successfully, with y^{-2} seen fairly often. A small number of students did not attempt to rearrange at all, but attempted direct integration, and these invariably made no progress.

Once variables were separated the left hand side was generally dealt with successfully, the method usually even scored if the incorrect separation noted above was seen.

The right hand side posed greater problems, but many successfully applied the double angle identity leading to an expression of the form $At^2 (1 + \cos 2t)$, though a few made errors in the double angle identity (usually with a subtraction of terms, occasionally using sin instead of cos). However, a few did not proceed thus at all, but attempted integration by parts, usually thinking $\cos^2 t$ integrates to $A\cos^3 t$.

Most who applied the double angle identity separated the integral to $A\int t^2 \, dt + A\int t^2 \cos 2t \, dt$ and recognised the connection with (a), though a significant amount started the integration by parts process again, sometimes reaching the correct form, sometimes not. The students who spotted the connection with part (a) were usually successful in scoring remaining method marks, though many failed to include the '+c' term, losing the final mark. Students usually remembered to integrate the t'2' term, but this was occasionally forgotten, losing the final method mark also.

Question 6

Vectors questions are often unliked by students, but in this question parts (a), (b) and (c) provided good access and were generally answered well, with part (d) being discriminating and only the better students able to make progress. On the whole, the responses to this question on vectors showed some improvement over previous years and there were far fewer blank scripts.

Part (a) was well answered by most students. Almost all students were able to use the vector line equations to extract at least two equations and attempt to solve them simultaneously for the parameters λ or μ (usually both). There were occasional slips in copying down an equation or in solving the simultaneous equations. Although many students found correct values for the parameters, which they were able to substitute correctly to find the value of p, a significant number of students used some very long, circuitous approaches, usually by a substitution method involving fractions and unsimplified expressions for their parameters. These students often made errors in manipulation or arithmetical slips, leading to an incorrect value for p. There were occasional sign errors, particularly in finding p. At least 3 (the method marks) out of the 4 marks were gained for most solutions.

For part (b) most students realised they needed to use one of the parameters found in part (a) to find the point of intersection. Most students were able to use $\mu = 2$ to find the correct coordinates or position vector of the point of intersection. A significant number used l_1, even though this meant using their values of λ and p, and so were more likely to lose the accuracy mark due to their values having been found incorrectly in part (a). Errors sometimes occurred by substituting into incorrect expressions which were likely to have been achieved by mixing up the components of the two lines l_1 and l_2.

There were a significant number of students who did not know how to proceed and left this part blank.

Students were very successful in part (c) in identifying that they needed to use the scalar product of two vectors to find the angle between them with the application of the formula to find an angle well demonstrated. However, use of incorrect direction vectors was fairly common, often using position vectors especially that of the point of intersection found in (b). Those who applied a correct method with correct vectors generally gained all 3 marks. A few rounded the size of the acute angle found incorrectly, often giving the 2 significant figure answer, and lost the accuracy mark, or gave the obtuse angle between the vectors. Very few students worked in radians.

Although part (d) was quite a challenging part, many students made some progress and there were a good number of fully correct answers. The majority of students were able to find **OA**, often using λ=2 as required, with their value of p and gained the B1ft. This was often the only mark gained, though. Though many students did realise they needed to take a scalar product to find the co-ordinates of B, some used **OB** in terms of μ rather than the direction vector **AB**. It was quite common to see **OA**, the position vector from l_2 or another vector in terms of μ, instead of the appropriate direction vector. Of those who did find **AB** in terms of μ, many would proceed to find the scalar product and the correct value for μ and then the correct coordinates for B. Sometimes there were sign and/or arithmetical errors in finding μ leading to an incorrect value, and a small number of students substituted μ into the wrong equation to achieve the co-ordinates of B. Very few students attempted the alternative approach which used the right-angled triangle; those that did generally made little progress.

Question 7

At this stage of the paper responses became a bit more varied, some completing the question with little trouble, but many lacking in the skills to suitably show the result in (a), or failing to make the connection between the two parts.

The majority of students made some attempt to start part (a), with many successfully gaining the first two marks for the correct differentiation of u, though there were some errors in the coefficient of cos 2x, making further progression difficult. The most successful responses used the double angle formula before substitution, getting $4(1 + \cos 2x) = 4(2\cos^2 x)$. Some clearly recognised that $1 + \cos 2x$ was $2\cos^2 x$, but many showed their working more clearly, having an intermediate line $4 + 4(2\cos^2 x - 1)$. It was common to see the substitution of $\mathrm{d}x = \frac{1}{8\cos^2 x}\mathrm{d}u$ into the given integral before cancelling the $\cos^2 x$, others simply replaced $\cos^2 x\,\mathrm{d}x$ by $\frac{1}{8}\mathrm{d}u$.

A small number of students used the double angle formula before differentiating u. In such cases success was usually only seen when $u = 4x + 4\sin x \cos x$ was differentiated using the product rule to attain $\frac{\mathrm{d}u}{\mathrm{d}x} = 4 + 4\cos^2 x - 4\sin^2 x$. This enabled $8\cos^2 x$ to be attained very easily using $\sin^2 x = 1 - \cos^2 x$.

As the question was a ‘show that’, some degree of rigour and clarity was required, but a large number of students did not clearly show their use of the double angle formula, sometimes quoting it incorrectly, yet attaining the correct given answer nevertheless. A significant number of responses lost the final two accuracy marks as they went directly from the integral to the given answer with no sign of a correct form of the actual integral and often no limits in terms of u. Only a very small number of students reverted to x and used the original limits.

In summary, for a show that question students should be reminded that all details need to be clearly shown in order to justify the award of the marks.

Part (b) also showed a great variation in the success rate. Some students who had not tried part (a), or who had made little progress, were successful, but many did not even try this part. Many wrote the correct formula, $V = \pi \int y^2 \mathrm{d}x$, but made no further progress, usually due to being unable to see how to square *y*. Of responses that showed a correct expression for y^2, some failed to relate it to part (a), seeking means such as integration by parts, or another attempt at substitution, or other incorrect approaches. But the majority who attained $36e^{4x + 2\sin 2x}$ did use the answer from part (a) correctly, with relatively few missing, or losing during working, the π.

Question 8

Many students made a fairly good attempt at this question and generally knew how to approach a proof by contradiction. Although few students scored no marks, some found it difficult to rigorously prove the contradiction, and attempts commonly gained just the first two marks.

The majority successfully set up their initial assumption, that there was a stationary point, and linked it to the fact that $\frac{dy}{dx} = 0$. Most then successfully differentiated the function, putting them in a position to find a contradiction. Instances of incorrect differentiation were rare.

However, many then simply commented that the equation formed had no solutions, giving no explanation as to why, or they gave an inadequate justification, and so gained no further credit.

Students who more successfully established a contradiction generally based their arguments on the facts that $x^2 \geq 0$ and $-1 \leq \sin x \leq 1$, or variations of these, justifying the required result either algebraically or in words, possibly with the aid of a diagram - though a diagram alone was not sufficient.. Other inappropriate methods on the equation, such as attempting the ‘discriminant’, using a graph only, testing values of *x* or attempting to use small angle approximations, were also seen, but unable to score the second M.

Generally, students who did successfully reach the contradiction were able to state the appropriate conclusion to complete the proof, but sometimes concluded that there were no stationary points without making it clear that there was a contradiction or did state there was a contradiction but failed to refer back to no stationary points and so lost the accuracy mark.

Most students found this question a challenge. Few gained full marks.

Question 9

For the final question on the paper, this was well attempted with many fully successful responses seen, and no evidence of a shortage of time to complete the paper was noted. However, students did not always take the most direct routes to the answers.

In part (a) most students were able to differentiate sec t accurately, indeed it is a standard formula in the formula book, with a few reverting to an expression in sine and cosine first, though these too were usually successful. However, when not correct sec t.cos t was the common answer.

More errors were seen when attempting to differentiate $\sqrt{3}\tan\left(t+\frac{\pi}{3}\right)$ especially in those who needlessly decide to apply the compound angle formula and apply the quotient rule - seldom were such attempts successful. Extra multiples of t or of $\frac{\pi}{3}$ were sometimes seen in those who recognised tan differentiates to sec^{2}., but the majority were able to differentiate both terms successfully.

Students, almost invariably, were able to apply the chain rule with their $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of t, though a few did divide the wrong way round, or multiplied instead. Although subsequent work after a correct response was ignored, a rather common mistake made by students was to "cancel" sec in numerator and denominator, thus having no regard for the different arguments involved.

Part(b) was tackled well, with a correct equation - in the required format - for the tangent being obtained by many students. The B mark for the coordinates of P was very often able to be awarded, and if not the y coordinate was usually the one causing error. Likewise, many were able to follow through their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and find the gradient of the curve at $t=\frac{\pi}{3}$. Some lost this mark, however, by not taking their work through to find a *numerical* value for the gradient, while others showed no working and so forfeited the mark for an incorrect value for their gradient function. Most could apply the formula for the equation of the straight line (or find the value of c) using their coordinates and gradient, to obtain the equation of the tangent in $y = mx + c$ format. However, a few instead found the normal, while various elementary errors in manipulating the algebra at this stage were also common.

Part (c) was a very different matter, providing somewhat of a challenging end to the paper, with a fair proportion making little or no attempt at this final section of the question. Yet most scored at least the first mark, and good progress was seen by many and the correct answer being obtained was not infrequent. For those who did tackle this final part of the question, most were able to expand $\tan\left(t+\frac{\pi}{3}\right)$ (some having done so in part (a)), and many were able, through use of the identity $\sec^2 t = 1 + \tan^2 t$, to obtain an equation for y in terms of x. Getting a correct equation was common, but the final two marks involved some challenging algebraic manipulation (involving rationalisation of the denominator) to get their expression into the format required, and only the most able were successful. A small number resorted to writing the tangent as y in terms of sine and cosine first, but likewise proceeded to apply the compound angle formulae and using appropriate identities to get a Cartesian equation linking y and x, but these also struggled to reach the correct form.

There were a very small number of students who focussed on finding value for A and B, by using points on the curve and solving simultaneous equations, for example, but did no work to justify the correct form of the equation. Such attempts could gain no marks unless they did equivalent work of expanding the compound and formula, and applying appropriate identities, to first reach an equation in tangents only, at which point it would be possible to proceed via equivalences to justify the form, though this was not successfully observed, with those taking such an approach settling on deducing the values for A and B only.